Math 2110Q Worksheet 14 Solutions
November 2, 2016

1. Find the volume of the region bounded by the surfaces $z = -1$, $z = 2$ and the hyperboloid $x^2 + y^2 - z^2 = 1$. *Hint: think about the curves on the surface of the hyperboloid for fixed z-values.*
Solution: On the surface of the hyperboloid, using cylindrical coordinates we have $x^2 + y^2 = r^2 = 1 + z^2$, so that $0 \le r \le \sqrt{1+z^2}$. The volume is therefore given by

$$\int_{-1}^{2} \int_{0}^{2\pi} \int_{0}^{\sqrt{1+z^2}} r\, dr\, d\theta\, dz = \pi \int_{-1}^{2} (1+z^2)\, dz = 6\pi.$$

2. Find the volume of the region bounded between the surfaces $z = 4 + x^2 + y^2$ and $z = 1 + 4x^2 + 4y^2$.
Solution: The surfaces intersect where $4 + x^2 + y^2 = 1 + 4x^2 + 4y^2$, or where $x^2 + y^2 = 1$. This means that the entire volume lies over the unit circle in the xy-plane, and z is bounded between the two surfaces, so that the volume is given by (cylindrical coordinates)

$$\int_{0}^{2\pi} \int_{0}^{1} \int_{1+4r^2}^{4+r^2} r\, dz\, dr\, d\theta = 2\pi \int_{0}^{1} 3r - 3r^3\, dr = \frac{3\pi}{2}.$$