

Math 1060Q Lecture 6

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Today we explore some special functions.

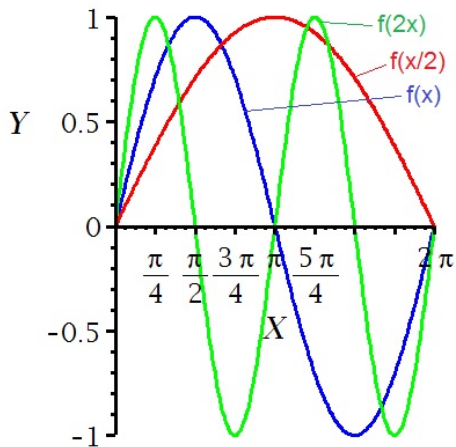
- ▶ Shifts, stretches and flips of graphs
- ▶ Absolute value function
- ▶ Square root function
- ▶ Greatest integer function
- ▶ Domain and range problems

Recall from last lecture: shifts of graphs and vertical stretches

- ▶ $f(x) + c, c > 0$: push graph “up” c units
- ▶ $f(x) - c, c > 0$: push graph “down” c units
- ▶ $f(x + c), c > 0$: move graph to left c units
- ▶ $f(x - c), c > 0$: move graph to right c units
- ▶ $cf(x)$: shrink ($0 < c < 1$) or stretch ($1 < c$) vertically by a factor of c

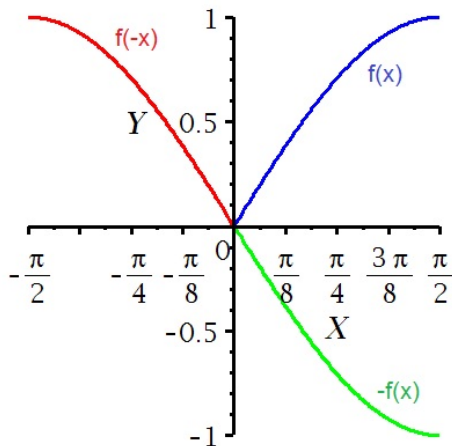
We can shrink or stretch horizontally by rescaling the argument

- ▶ $f(cx)$, $0 < c < 1$: stretch horizontally
- ▶ $f(cx)$, $c > 1$: shrink/compress horizontally



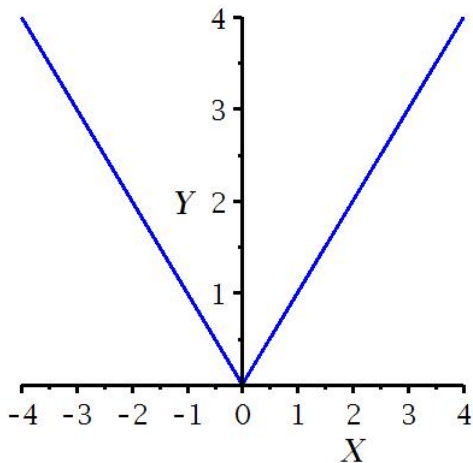
We can flip across the x -axis or the y -axis as well

- ▶ $f(-x)$: flip across y -axis
- ▶ $-f(x)$: flip across x -axis

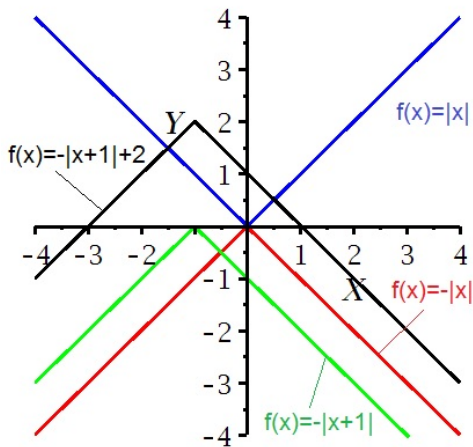


- ▶ Shifts, stretches and flips of graphs
- ▶ **Absolute value function**
- ▶ Square root function
- ▶ Greatest integer function
- ▶ Domain and range problems

The basic shape of $f(x) = |x|$ is a “V”. Note the domain and range.

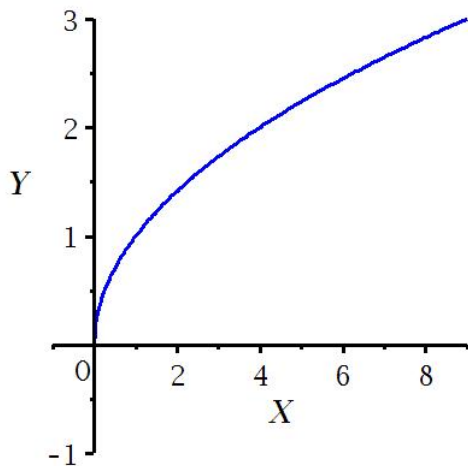


Now we can manipulate the graph via shifts, flips and stretches

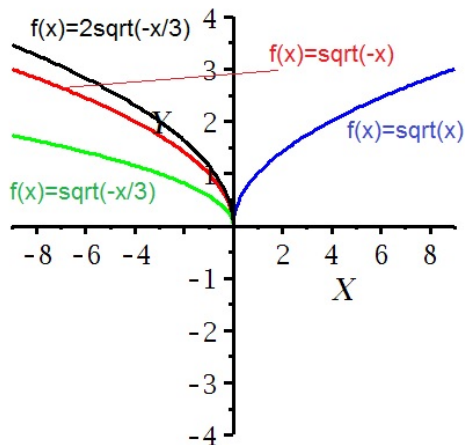


- ▶ Shifts, stretches and flips of graphs
- ▶ Absolute value function
- ▶ **Square root function**
- ▶ Greatest integer function
- ▶ Domain and range problems

The square root function: note the shape, domain, range.

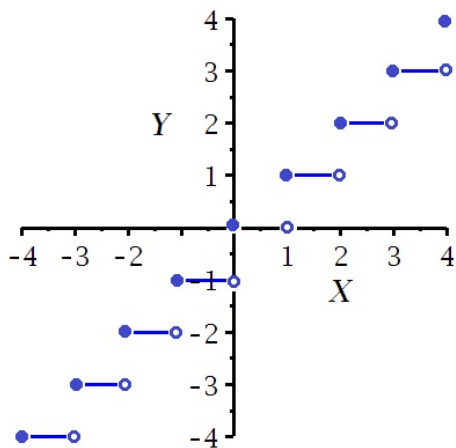


Manipulations of the square root function

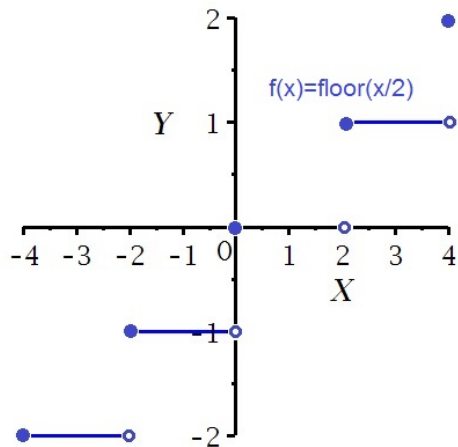


- ▶ Shifts, stretches and flips of graphs
- ▶ Absolute value function
- ▶ Square root function
- ▶ **Greatest integer function**
- ▶ Domain and range problems

The greatest integer or “floor” function: note domain and range.



What is interesting is to stretch horizontally.



- ▶ Shifts, stretches and flips of graphs
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You will want to be able to answer questions as in these examples.

Example L6.1: What are the domain and range of

$$f(x) = \frac{1}{2}\sqrt{x-1}?$$

Solution: You need what is under the “radical” to be non-negative:

$$x - 1 \geq 0 \Rightarrow x \geq 1.$$

Therefore, the domain is $[1, \infty)$. For the range, the coefficient $\frac{1}{2}$ does not matter. There is no vertical shift; the range is $[0, \infty)$, just like for \sqrt{x} .

Example L6.2: What are the domain and range of

$$f(x) = |2x + 1| - 1?$$

Solution: You should think of this as $f(x) = |2(x + 1/2)| - 1$. This function is generated from $|x|$ by shifting left $1/2$ unit, contracting horizontally by a factor of 2 and shifting down one unit. Therefore, the domain is still \mathbb{R} and the range is $[-1, \infty)$.

Similar examples.

Example L6.3: What are the domain and range of

$$f(x) = \text{floor}(x - 1/3) - 1?$$

Solution: Shifting horizontally does nothing to affect the domain, which is \mathbb{R} . Shifting vertically by an integer does not affect the range; it is all integers: \mathbb{Z} .

Example L6.4: What are the domain and range of

$$f(x) = \frac{1}{2}\text{floor}(x)?$$

Solution: The domain is the same as before; the range is now:

$$\mathcal{R} = \left\{ x \mid x = \frac{k}{2}, k \text{ is an integer} \right\}.$$

Practice!

Problem L6.1: Sketch the graph of $f(x) = \sqrt{x+2} - 3$.

Problem L6.2: Sketch the graph of $f(x) = -|x-3| + 1$.

Problem L6.3: Sketch the graph of $f(x) = \text{floor}(x+1)$.

Problem L6.4: Sketch the graph of $f(x) = |5x-5|$.

Problem L6.5: Find the domain and range of $f(x) = -2\sqrt{x+4}$.

Problem L6.6: Find the domain and range of $f(x) = |x| + 3$.